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DETERMINATION OF THE BEHAVIOR OF A STANDARD LINEAR BODY IN A PLASTOMERE OF PLANE-PARALLEL SHEAR

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Consider the behavior of a plastomere containing an easily deformed material, whose local mechanical properties are described by a model of a standard linear body [1-4], represented by three physical quantities: the viscosity η, the instantaneous shear modulus G_0, and the long-term shear modulus μ.

A plastomere, intended for investigating nonflowing materials, is shown schematically in Fig. 1, where 1 is a steel plate rigidly connected to the instrument, 2 is the specimen of thickness l, 3 is the sheared rigid plate, 4 is an indicator rod rigidly attached to the sheared plate, 5 is a retainer for fixing the initial state of the system and for producing the required initial conditions of motion of the plate, 6 is a pulley with a reduced moment of inertia I and external radius R, M_0 is the reduced mass of the sheared plate and the attached rod, P_1 and P_0 are the loads attached to the sheared plate, u(x, t) is the displacement function of an infinitely thin vertical layer of the material investigated in the direction in which the external forces of plane-parallel shear act, x is the axis of coordinates, and t is the time. The load P_0 is connected to the plate with a steel wire. The specimen used has a rigidity much less than the rigidity of the instrument parts [5]. In the initial state the position of the rod is held rigidly by means of the retainer. At the instant of time t = 0 the upper end of the rod is released from the retainer and the plate begins to move under the action of the resulting load $P = P_1 + M_0g - P_0$. Then the connecting mass [5] $M = M_0 + (P_1 + P_0)/g + I/R^2$, where P_1/g, P_0/g, and I/R^2 are the reduced masses of the loads P_1 and P_0 and of the rotating pulley, and g is the acceleration due to gravity. It is assumed that the force of friction f in the bearings of the pulley is much less than the load P, so that it can be neglected [5]. This system differs considerably from the plastomere used in [5-7]. Its distinguishing features are as follows.

1. Since the shear load is the complex quantity $P = P_1 - P_0 + M_0g$, and $M = M_0 + (P_1 + P_0)/g + I/R^2$, for the same values of I, R, and P, one can increase or decrease the value of M over a wide range by varying P_1 and P_0 while keeping P constant. This enables one to investigate materials both when the system is oscillating and under aperiodic conditions.

2. The system enables one to eliminate f by removing the load P_0.

Since the above viscoelastic characteristics of easily deformed materials may depend, in particular, on the temperature T [4] and this may cause deformation of the specimen, the experimental conditions must be

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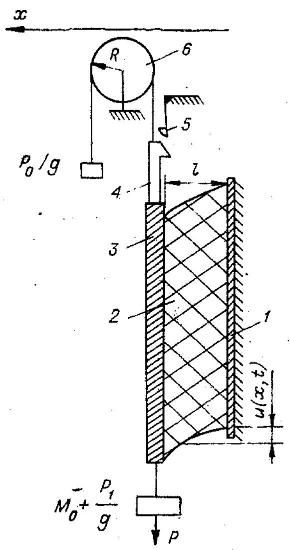

Fig. 1

chosen to be such that one can assume the temperature T and each of the characteristics η, G_0, μ, τ to be the same over the whole volume of the specimen of a uniform material. We will choose the deformation conditions of the specimen to be such that one can write the differential relation between the local relative deformation γ and the plane-parallel shear stress σ in the form

$$\sigma + \tau\dot{\sigma} = \mu\gamma + G_0\tau\dot{\gamma}, \tag{1}$$

where $\tau = \eta/(G_0 - \mu)$ is the relaxation time and $G_0 > \mu$.

Suppose the length of the specimen and the plate of the plastomere are sufficiently large compared with the distance between the plates l, and the external shear load is small, so that the maximum value of the stress does not exceed the critical value for which plastic flow begins. We can then assume that the microstructure of the specimen remains unchanged, and the deformation takes the form of a plane-parallel shear. Suppose also that the rigidity of the specimen is much less than the rigidity of the parts of the plastomere. We will neglect the deformations of the latter and assume that all the elements of the reduced mass M are displaced in time with the same speed and acceleration, remaining parallel to one another. Then, as in [5], we can assume that the mass M is concentrated in an infinitely thin layer on the sheared boundary $S(l)$ of the specimen, where $S(l)$ is the area of contact of the "sheared plate-specimen." Due to the shear force produced by the load P on the surface $S(l)$ as $\eta \to \infty$, a plane-parallel shear stress occurs in the form (2) of [5]. When $f \ll P$ the value of f can be neglected, and when $P \gg f$ it is necessary to remove the load P_0 so that the value of f is eliminated from the relation of the form (2) in [5]. In the latter case $M = M_0 + P_1/g$. All the other necessary assumptions and approximations are the same as in [5]. The main one of these is the assumption that the deformation process is isothermal and that the temperature over the whole specimen is the same. With these assumptions and approximations the motion of the system is described by the function $u(x, t)$ which gives the value of the vertical displacement of an infinitely thin layer of material at a distance x from the fixed boundary of the specimen, and the relative shear of this layer is given by the quantity $\gamma = \partial u(x, t)/\partial x$ (Eq. (3) of [5]). Using relation (1) we can write the differential equation for finding the function $u(x, t)$ in the form

$$\tau\rho u_{ttt} + \rho u_{tt} = \mu u_{xx} + G_0\tau u_{txx}, \tag{2}$$

where ρ is the density of the deformed material. We will further assume that when $t < 0$ the upper end of the rod is fixed rigidly by the retainer and the value of the external load on the boundary $S(l)$ of the specimen is $P = 0$, and when $t \geq 0$ the upper end of the rod is unlocked and the external load $P = P_1 + M_0 g = \text{const}$ (or $P = P_1 + M_0 g - P_0$ when $P \gg f$). Then the two initial conditions of the problems can be represented in the form

$$u(x, 0) = 0, \; u_t(x, 0) = 0 \quad \text{for} \quad 0 \leqslant x \leqslant l. \tag{3}$$

We take as the third initial condition the relations [5]

$$\rho u_{tt}(x, 0) = G_0 u_{xx}(x, 0) \quad \text{for} \quad 0 \leqslant x < l; \tag{4}$$

$$SG_0 u_x(l, 0) = P - Mu_{tt}(l, 0) \quad \text{for} \quad x = l. \tag{5}$$

The boundary conditions of the moving mass of the specimen when $t > 0$ can also be written in the form [3]

$$S[\mu u_x(l, t)+G_0\tau u_{xt}(l, t)] = P - M[\tau u_{ttt}(l, t)+ u_{tt}(l, t)], \quad u(0, t)= 0. \tag{6}$$

Relations (4)-(6) show that at the initial instant when the load $P(t=0)$ acts the specimen behaves as an idealized elastic system, and when $t > 0$ it acts as a viscoelastic system. Thus at the instant $t=0$ the system is in its initial natural state [1, 2], and due to the action of the load P at any instant of time $t \geq 0$ the stress and deformation in the specimen propagate with a finite velocity not exceeding the velocity of sound [8], and the relaxation processes due to the resistive forces of internal friction are retarded with respect to the instants of propagation of the stress in the deformed distributed mass. In view of these physical properties, the nature of the redistribution of the forces at the boundary $S(l)$ of the specimen can be expressed by a condition of the form (5) when $t=0$, and by an expression of the form (6) when $t > 0$, while inside the specimen and close to the boundary $S(l)$ it is expressed by expression (4) when $t=0$, and by (2) when $t > 0$, respectively.

All the coefficients for the partial derivatives in (2)-(6) are assumed to be constant. Equation (2) and (3)-(6) differ from the problem considered in [5] by the presence of terms with the coefficient μ and the quantity P_0.

In addition, whereas when solving the problem for a Maxwellian body the third-order initial equation in time t (see (4) in [5]) can be converted mathematically into a second-order equation with nonuniform initial conditions, while the solution of the problem can be simplified and reduced mathematically in the final analysis to the solution of a second-order differential equation without singularities of the type (4) and (5) (see (6) and (7) in [5]), in this case Eq. (2) does not lend itself in principle to this mathematical transformation, and when determining the transient parts of the required displacement function of the specimen, singularities of the type (4) and (5) remain, and effect the process and the result of the solution of our problem. Note that a similar problem but with different initial and boundary conditions is considered in [2, 3].

Separating the variables in Eqs. (2)-(6), the solution of problem (2) can be written in the form

$$u(x, t) = vx + 4ve\sum_{k=1}^{\infty} \frac{\sin\beta_k \sin\beta_k \frac{x}{l}}{\beta_k(2\beta_k+\sin 2\beta_k)}\, e^{-\frac{t}{3\tau}} \left[\frac{C_k e^{\alpha_k t} + e^{-\frac{\alpha_k}{2}t}(M_k\cos w_k t + N_k \sin w_k t)}{3\alpha_k^2 + p_k}\right], \tag{7}$$

where β_k is the positive solution of the equation

$$\operatorname{ctg}\beta = M\beta/\rho l S. \tag{8}$$

Here we have introduced the following notation:

$$\begin{gathered}
C_k = (G_0-\mu)\beta_k^2/\rho l^2 + 2/9\tau^2 - \alpha_k/3\tau - \alpha_k^2,\\
M_k = 1/9\tau^2 + \alpha_k/3\tau - 2\alpha_k^2 - \beta_k^2\mu/\rho l^2,\\
N_k = \frac{1}{w_k}\left[\alpha_k\left(p_k + \frac{1}{6\tau^2} - \frac{3\mu\beta_k^2}{\rho l^2}\right) - \frac{\alpha_k^2}{\tau} - \frac{p_k}{3\tau}\right],\\
\alpha_k = v_k - p_k/3v_k, \quad v_k = \sqrt[3]{-q_k/2 + \sqrt{D_k}},\\
w_k = \sqrt{3}(v_k + p_k/3v_k)/2, \quad p_k = G_0\beta_k^2/\rho l^2 - 1/3\tau^2,\\
q_k = (\mu - G_0/3)\beta_k^2/\tau\rho l^2 + 2/27\tau^3, \quad v = P/S\mu,\\
D_k = p_k^3/27 + q_k^2/4 > 0.
\end{gathered} \tag{9}$$

An expression of form (7), taking (8) into account, only holds when $D_k > 0$ in (9). When $D_k < 0$ expression (7) may contain aperiodic solutions of the equation for the transient part of the required function $u(x, t)$. To determine the limits of applicability of (7) it is best to convert D_k in (9) to the form

$$D_k = \{y_k^2 - by_k/4 + a^3\}\,\mu^3\beta_k^6/27\rho^3 l^6, \tag{10}$$

where

$$\begin{gathered}
y_k = \rho l^2/\tau^2\mu\beta_k^2;\\
b = a^2 + 18a - 27; \quad a = G_0/\mu.
\end{gathered} \tag{11}$$

The roots of the trinomial are $y_k^{(1)} = b/8 - \sqrt{d}$, $y_k^{(2)} = b/8 + \sqrt{d}$, where $d = (a-1)(a-9)^3/64$. The condition for D_k in (9) to be positive is satisfied in the following cases $y_k < y_k^{(1)}$ when $d > 0$ or $y_k > y_k^{(2)}$ when $d > 0$, and $y_k \neq y_k^{(1)} = y_k^{(2)} = b/8$ when $d = 0$ and for all y_k when $d < 0$ $(1 < a < 9)$. (We will henceforth assume $a > 1$, which is always satisfied for the model chosen).

If $a > 9$, in which case y_k lies in the range

$$b/8 - \sqrt{d} < y_k < b/8 + \sqrt{d}, \tag{12}$$

then $D_k < 0$. Hence, Eq. (2) in this case when $a > 9$ may contain the above-mentioned aperiodic solutions in the sum of the series of transient functions in an expression of the form (7). We will consider the case when such solutions exist. Suppose $Sl\rho/M \ll 1$, which is partially realized in practice. Then, the roots of Eq. (8) can be represented in the form [8]

$$\beta_1 \cong \sqrt{\rho l S/M}, \quad \beta_k \cong (k-1)\pi + 2\rho l S/M.$$

We will assume that one of them, e.g., β_1, which occurs in relation (11), satisfies the inequality $y_1 > b/8 + \sqrt{d}$ when $a > 9$. Then, one can obtain the values of y_k lying in the range (12) and thereby giving rise to the presence of the above-mentioned aperiodic solutions in the sum of the series of the form (7). These situations can be expected when investigating the behavior of rubber in the plastomere [4].

We will show that when elastomers (which are in a highly elastic state) are deformed in a plastomere, the transient function of the solution of the form (7) may actually contain aperiodic solutions. To do this we will write the inequalities (12) in terms of the actual parameters of the specimen and the apparatus used. We will take a specimen of length $h = 10$ cm, $l = 0.4$ cm, $S = 20$ cm^2, $P_0 = 10^6$ dyne, $P = 3 \cdot 10^4$ dyne, $\eta = 10^3$ P, $G_0 - \mu = 3 \cdot 10^5$ dyne/cm^2, $\mu = 3 \cdot 10^3$ dyne/cm^2, $\rho = 1$ g/cm^3, and $I/R^2 = 40$ g. Then $M = 2.07 \cdot 10^3$ g. We will express inequality (12) for the first terms of the sum of the series of the expansion of the transient part of the function (7) with parameter $\beta_1 \cong \sqrt{\rho l S/M} \ll 1$ in the form

$$(G_0^2/\mu^2 + 18G_0/\mu - 27)/8 - \sqrt{(G_0/\mu - 1)(G_0/\mu - 9)^3/64} < \rho l^2/\tau^2 \cdot \mu\beta_k^2 <$$
$$< (G_0^2/\mu^2 + 18G_0/\mu - 27)/8 + \sqrt{(G_0/\mu - 1)(G_0/\mu - 9)^3/64}.$$

Substituting into this equality the values of the parameters employed (with the approximations $\sqrt{(G_0/\mu - 1)} \times \sqrt{(G_0/\mu - 9)^3/64} \cong 1104$ and $M \cong 2 \cdot 10^3$ g), we obtain $y_1 \cong 1.2 \cdot 10^3$, $y_k^{(2)} \cong 2.6 \cdot 10^3$, $y_k^{(1)} \cong 395$, i.e., $395 < y_1 < 2.6 \cdot 10^3$.

If we substitute into (12) the parameter $\beta_2 \cong \pi + 2\rho l S/M$, then y_2 turns out to be less than 395, i.e., $y_2 \cong 1.6$. Hence, under the experimental conditions employed the first terms with β_1 in the sum of the expansion in series of the transient part of the function of the form (7) represent aperiodic motion of the deformed mass, while the terms following them with $\beta_2, \beta_3, \beta_4, \ldots$ represent oscillatory motion.

Hence, when determining the physical quantities η, G_0, μ, and τ and also when setting up and processing programs for calculating them on a computer using equations of the form (7) one must bear in mind all kinds of situations that can arise when $a > 9$ and $y_1 > b/8 + \sqrt{d}$ (or $y_1 > b/8 - \sqrt{d}$). The solution of the form (7) when $\mu = 0$, as might have been expected, reduces to the result obtained in [5]. If we put $M = 0$ in (7) and pass to the limit as $\eta \to \infty$, Eq. (7) agrees with the solution of the analogous problem for an idealized elastic system [5]. Since in our case the mass $M = (P_1 + M_0 g)/g$ (or $P_1/g + P_0/g + M_0$, when $P \gg f$), the limit of expression (7) as $\eta \to \infty$ is also of interest when $M \neq 0$,

$$u(x, t) = \frac{\nu\mu}{G_0}\left\{x - 4l \sum_{k=1}^{\infty} \frac{\sin\beta_k \sin\beta_k \frac{x}{l} \cos\beta_k \sqrt{\frac{G_0}{\rho l^2}}\,t}{\beta_k(2\beta_k + \sin 2\beta_k)}\right\}. \tag{13}$$

Hence, the solution of form (7) is more general than that given in [5]. It can be used to describe the motion of a standard linear body, a Maxwellian body, and also to investigate materials with high viscosity ($\eta \to \infty$).

It is seen that (7) contains the main local (including the viscoelastic characteristics of the specimen (ρ, G_0, μ, η, τ) and the technical characteristics of the instrument and the external load (P, M, S, l, y_k, α_k, w_k) in their extremely complex relationships. In this connection Eq. (7), which is a function of the creep, is specific and for this reason is suitable for describing specific experimental situations. The solution of form (7) can also be used as one of the initial functions both when determining the viscoelastic properties of specimens and for predicting the mechanical behavior of viscoelastic materials that are easily deformed under shear, which occurs in technical systems when small external concentrated loads are applied.

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STRENGTH OF SINGLE-LAYER AND MULTILAYER CYLINDRICAL VESSELS LOADED INTERNALLY BY PULSES OF VARIOUS LENGTHS

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Technological processes and experimental research involving the detonation of explosives must ensure the containment of the detonation products, the safety of personnel, and the protection of equipment. Frequently the test conditions impose rigid requirements on the size and weight of the vessels designed to localize the effect of pulsed loads of various kinds. Pressure vessels for operation under static loads, where the controlling parameter is the pressure, are commonly of multilayer construction. As a consequence of the stopping of cracks in the separate layers [1, 2] this construction can increase the level of the working pressure and avert the catastrophic rupture of the whole structure. It is of interest to investigate the behavior of multilayer shells under pulsed loads of various duration.

We have investigated the effect of an internal explosion on closed cylindrical vessels with an outside radius R_0 and a total wall thickness δ_0 (Fig. 1) filled with air at normal atmospheric pressure.

The difference in duration of the pulsed loading was achieved by different schemes for loading the vessel walls. In the first case (Fig. 1a) the vessel walls were loaded by detonation products, and in the second (Fig. 1b) by the impact of a thin auxiliary shell 3 accelerated by the detonation products.

The vessels were made of Kh18N10T austenitic stainless steel (as received) and had the same geometry but differed in the design of the casing 1. A single-layer casing was made of tubular stock (GOST 5632-61) and had one welded joint along a generator of the cylinder. The casing was joined to a seamless core 2 of thickness $\delta' = 2$ mm (Fig. 1) with no gap between them. A multilayer casing was made by winding five layers of millimeter sheet (1 GOST 3680-71) of width $4R_0$ onto the core as in [1] with negligible clearance between layers; the inside and outside ends of the sheet were welded. The covers of the vessels 4 were attached with twelve M 16 bolts. The initial mechanical properties of the materials of the single-layer and multilayer casings determined on samples of identical dimensions under steady tension at a strain rate of $5 \cdot 10^{-4}$/sec are listed in Table 1.

A spherical explosive charge of radius r (50 wt. % TNT, 50 wt. % hexogen, density 1.65 g/cm^3) was placed at the center of the vessel. The charge was fired from the center. All the experiments were performed at

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